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## ORIGINAL STUDY

# Elastic Scattering of Deuteron Beam on ${}^6\text{Li}$ Nuclei in the Context of the Cluster Model

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### Abstract

In the context of the cluster model applied to the nuclear reaction  $d + {}^6\text{Li}$  at a laboratory energy of 171 MeV, the nucleus  ${}^6\text{Li}$  was examined within the framework of an  $\alpha + p + n$  cluster model. The three-body model concept was incorporated into the interaction potential for the  $d + {}^6\text{Li}$  system through the construction of a double-folding potential based on the three-body model.

The computed double-folding potential was employed as the real component of the optical potential for analyzing the experimental differential cross-section of elastic scattering in the  $d + {}^6\text{Li}$  nuclear reaction at 171 MeV laboratory energy. Meanwhile, the imaginary component was chosen as the Woods-Saxon potential. The parameters of the optical potential were adjusted to optimize agreement with experimental data.

This study includes comparisons between the calculated semi-microscopic potential and a phenomenological potential, accomplished through differential cross-section analysis. Additionally, parameters of the potentials are presented in tabular form. The best agreement with experimental differential cross-sections was achieved using the computed folding potential, based on the three-body model, which clearly indicates the  $\alpha + p + n$  cluster structure within the  ${}^6\text{Li}$  nucleus.

**Keywords:** Elastic scattering, Optical model, Double folding potential

## 1. Introduction

The interest in  $p$ -shell nuclei remains a top priority in modern nuclear physics [1–5], especially with the development of secondary beam production, enabling research on nuclei beyond the stability line of the nuclear chart [6–8]. The nucleus  ${}^6\text{Li}$  holds a special position due to its unusual cluster structure [9–11]. This nucleus can be examined within the framework of the three-body model  $\alpha + p + n$ , and its characteristics can be observed both experimentally and theoretically.

From a theoretical perspective, Weinberg's pioneering efforts have made it possible to compute atomic nuclei using steadily improving Hamiltonians. Interestingly, artificial neural networks have shown effectiveness in representing nuclei with up to four

nucleons, and this method has been expanded to include  ${}^6\text{Li}$  and  ${}^6\text{He}$  nuclei using a simplified effective field theory Hamiltonian [12–14]. This AI-based technique accurately compares binding energies, point-nucleon densities, and radii with those calculated using the hyperspherical harmonics method.

The research from Ref. [15] investigates the differential cross-sections of elastic and inelastic scattering of deuterons on various target nuclei:  ${}^6\text{Li}$ ,  ${}^{16}\text{O}$ ,  ${}^{32}\text{S}$ ,  ${}^{50,51}\text{V}$ , and  ${}^{70,72}\text{Ge}$  at a laboratory energy of 171 MeV. The experimental data were obtained analyzed within the framework of the optical model, and their parameters were compared with global optical parameterizations. Furthermore, the inelastic scattering channels were analyzed using the coupled-channel method. In this context, a deformation parameter of  $\beta_2 = 0.89$  was selected for the nucleus of our interest.

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It is also noteworthy to mention another study [16] regarding the weakly bound nucleus  ${}^6\text{He}$ . The research investigated the elastic scattering of 151 MeV alpha particles on  ${}^6\text{He}$  nuclei [17]. In this investigation, the internal structure of the weakly bound nucleus was studied, considering it as an  $\alpha + n + n$  cluster structure. This concept was utilized in constructing the interaction potential between  $\alpha$ -particles and  ${}^6\text{He}$  nuclei using the three-body wave function [18]. Detailed descriptions of the elastic channel cross-sections were provided within the framework of the optical model. However, for a comprehensive description, including backward scattering angles, calculations based on the elastic transfer of two nucleons were employed. It was found that such behavior of the cross-section, namely, an increase in the cross-section at backward angles, is characteristic of nuclei possessing the  $\alpha + 2\text{N}$  structure.

Following the aforementioned studies, it is intriguing to pursue the following attempt to investigate the elastic scattering of deuterons on  ${}^6\text{Li}$  nuclei in order to probe the internal three-body cluster structure of  $\alpha + p + n$ . In this endeavor, we will utilize experimental data on the elastic scattering of deuterons on  ${}^6\text{Li}$  nuclei at an energy of 171 MeV [15]. Employing a three-body structure model [16,18], calculations will be conducted for the differential cross-section within the framework of the optical model for comparison with experimental data.

The article is structured as follows: after an introductory section, the second section is dedicated to the interpretation of the obtained results within the optical model. The concluding part of this work outlines prospective research directions pertaining to this topic and summarizes key findings.

## 2. Optical analysis

The scattering of deuterons on  ${}^6\text{Li}$  nuclei at an energy of 171 MeV were analyzed within the framework of the optical model method with the optical potential  $V(R)$ :

$$V(R) = V^C(R) + V^{SO}(R) + V^V(R) + iW(R), \quad (1)$$

where,  $R$  is the distance between the nuclei  $d$  and  ${}^6\text{Li}$ ,  $V^C(R)$  is the Columb potential, which can be represented as the interaction between the point-charge and uniformly charged sphere in the following form:

$$V^C(R) = \begin{cases} \frac{Z_d Z_{6\text{Li}} e^2 \left(3 - \frac{R^2}{R_C^2}\right)}{2R_C}, & \text{for } R \leq R_C \\ \frac{Z_d Z_{6\text{Li}} e^2}{R}, & \text{for } R > R_C. \end{cases} \quad (2)$$

Here,  $Z_d$  and  $Z_{6\text{Li}}$  are the charge numbers of  $d$  and  ${}^6\text{Li}$ ,  $R_C$  is the free parameter. The term  $V^{SO}(R)$  is the spin-orbit interaction, and its form can be given as follow:

$$V^{SO}(R) = V_0^{SO} \left(\frac{\hbar}{m_\pi c}\right)^2 \frac{1}{R} \frac{d}{dR} \left(1 + \exp\left(\frac{R - R_{SO}}{a_{SO}}\right)\right)^{-1} \quad (3)$$

$V^V(R)$  and  $iW(R)$  from Eq. (1) are the real and imaginary volume potentials.

The imaginary part of the optical potential  $iW(R)$  was taken as the first derivative of the Woods-Saxon potential. Using the derivative of the Woods-Saxon potential for the imaginary part of the optical potential is motivated by the need to accurately model the absorption of the projectile at the nuclear surface. In nuclear reactions, absorption is most significant near the surface of the nucleus. This is because at the nuclear surface, the projectile is most likely to interact with nucleons in a way that leads to inelastic scattering or other reaction channels.

The real part  $V^V(R)$ , in this work, was chosen in two forms: the Double folding (DF) and phenomenological Woods-Saxon (WS) potentials:

$$V^V(R) \equiv \begin{cases} V^{DF}(R) & \text{for DF} \\ V^{WS}(R) & \text{for WS.} \end{cases}$$

In this work, the  $V^{DF}(R)$  potential for the  $d + {}^6\text{Li}$  system was calculated using the nuclear matter density distribution function of the  ${}^6\text{Li}$  nucleus [19]:

$$V^{DF}(R) = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \rho_1(\mathbf{r}_1) \rho_2(\mathbf{r}_2) V_{NN}(\mathbf{R} - \mathbf{r}_1 + \mathbf{r}_2), \quad (4)$$

here,  $\mathbf{r}_1$  and  $\mathbf{r}_2$  are the internal radii of the  $d$  and  ${}^6\text{Li}$  nuclei,  $\rho_1$  and  $\rho_2$  are the nuclear matter density functions of  $d$  and  ${}^6\text{Li}$ , respectively, and  $V_{NN}$  is the nucleon-nucleon interaction, which is taken in the form of a sum of Yukawa potentials with parameters taken from Ref. [20].

In turn, the density function of  ${}^6\text{Li}$  was constructed on the basis of the three-body wave function:

$$\rho_2(R) = \sum_{i=ijk} \rho_i(R). \quad (5)$$

where  $ijk$  indicates the particles in the three body system, the  $\rho_N$  and  $\rho_\alpha$  are the nuclear matter density functions of nucleon and alpha particle inside  ${}^6\text{Li}$ :

$$\rho_N(R) = \langle \varphi^\gamma(i, jk) | \delta(\mathbf{R} - \mathbf{y}_i) | \varphi^\gamma(i, jk) \rangle, \quad (6)$$

$$\rho_\alpha(R) = \langle \varphi^\gamma(i, jk) | \rho_\alpha(\mathbf{R} - \mathbf{y}_i) \delta(\mathbf{y}_i - \mathbf{r}_\alpha) | \varphi^\gamma(i, jk) \rangle. \quad (7)$$

Here,  $\varphi^\gamma(i, jk)$  is the basis function for the nuclear model  $\alpha + p + n$ , obtained within the framework of

variational calculus [18]. In this case, the basis function for the three-body wave function is a multidimensional Gaussian function.

The nuclear matter density function for the deuteron  $\rho_1$  is selected as the Gaussian shape fitted to its *rms* radius.

The parameters were selected to obtain the best reproduction of the experimental cross sections. As a result of minimization  $\chi^2$  the best value  $\chi^2/N$  was obtained 0.6. The parameters of the imaginary part, such as depth, radius and diffuseness, correspond to the values 1.25 MeV, 3.784 fm and 0.484 fm. The spin-orbit part also has its contribution to the behavior of the cross section, and its similar parameters turned out to be 1.39 MeV, 1.332 fm and 0.544 fm.

In Fig. 1 compares the experimental cross sections with two calculated cross sections: *DF*, based on the three-body wave function, and *WS*, the phenomenological potential taken from Ref. [15]. Both potentials describe these cross sections perfectly, judging by the values  $\chi^2/N$  for potentials *DF* and *WS* 0.68 and 2.37 [15]. It is worth noting here that the potential *DF* was able to describe the data a little better, even with only 7 free parameters, while the *WS* potential uses 12 parameters.

In Fig. 2 graphs of the real part of the optical potentials are plotted. Potentials do not have a similar shape. However, in the region between 1.5 fm and 2.5 fm there is an intersection of functions, which indicates the location of the interaction radius for the  $d + {}^6\text{Li}$  system at the laboratory condition of 171 MeV. The *DF* potential turned out to be less decaying.

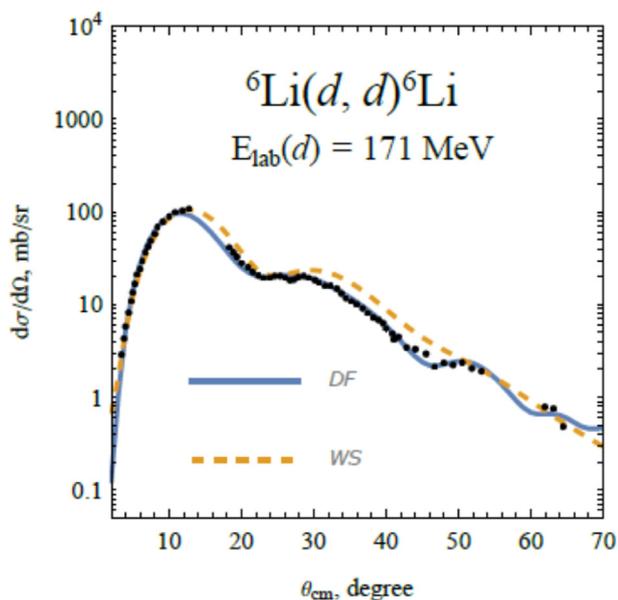


Fig. 1. Differential cross sections in comparison of the experimental data [15] with the optical model calculations: *DF* and *WS* cross sections based on the *DF* and *WS* potentials respectively.

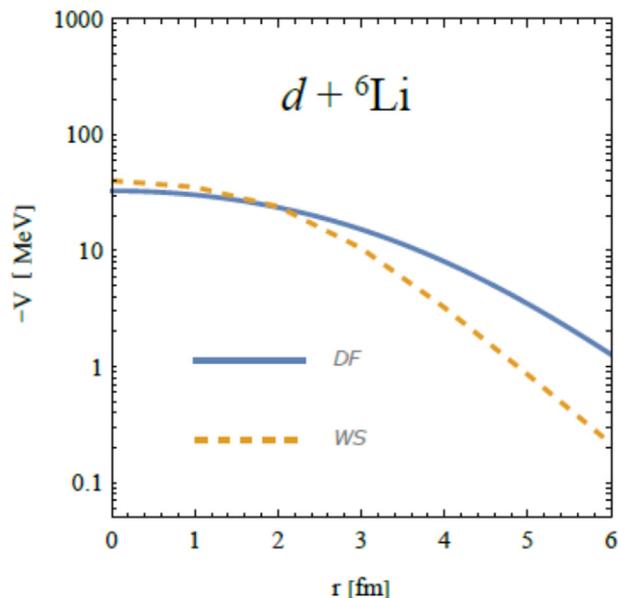


Fig. 2. Real part of the optical potentials: *DF* is based on the three-body model, *WS* is phenomenological Woods-Saxon potential.

This fact confirms the looseness of the internal structure of the  ${}^6\text{Li}$  nucleus, indicating its cluster structure.

Volume integrals of potentials are important characteristics that provide integral information about the interaction between scattered particles and the target nucleus. For the double folding potential *DF* and the phenomenological Woods-Saxon potential *WS*, the volume integrals show differences in the description of the interaction of deuterons with the  ${}^6\text{Li}$  nuclei.

In the case of the *DF* potential, the calculated volume integral is  $380 \text{ MeV} \cdot \text{fm}^3$ . This potential is based on the three-body wave function for the cluster structure  $\alpha + p + n$  of the  ${}^6\text{Li}$  nucleus, which allows for more accurate consideration of the features of the internal structure of the nucleus. The high value of the volume integral for the *DF* potential indicates more extensive interaction between the deuteron and the  ${}^6\text{Li}$  nucleus, which is consistent with the hypothesis about the cluster nature of the nucleus.

For the potential *WS* taken from the source, the volume integral is equal to  $325 \text{ MeV} \cdot \text{fm}^3$ . This phenomenological potential is often used to describe interactions within an optical model based on empirical parameters derived from experimental data. Despite the slightly lower value of the volume integral compared to the *DF* potential, the *WS* potential still provides an acceptable description of the interaction, but may underestimate the contribution of cluster effects to the structure of the  ${}^6\text{Li}$  nucleus. The difference in the values of the volume integrals between the *DF* and *WS* potentials highlights the importance of taking into account

cluster effects for more accurate modeling of nuclear interactions.

### 3. Conclusion and outlooks

In this study, data on elastic scattering of deuterons on  ${}^6\text{Li}$  nuclei at an energy of 171 MeV were analyzed using an optical model. The real part of the optical potential was obtained from the double folding potential  $DF$  calculated using the three-body cluster model  $\alpha + p + n$ . The imaginary part of the optical potential has been described using the phenomenological Woods–Saxon  $WS$  potential.

The results showed that the  $DF$  potential provides a better fit to experimental data compared to the phenomenological  $WS$  potential. This is confirmed by the value of the volume integral for  $DF$  equal to 380 MeV·fm<sup>3</sup>, which indicates more extensive interaction between the deuteron and the  ${}^6\text{Li}$  nucleus. While the volume integral for the  $WS$  potential is 325 MeV·fm<sup>3</sup>, which somewhat underestimates the contribution of cluster effects to the structure of the  ${}^6\text{Li}$  nucleus.

To further deepen the understanding of the interactions of deuterons with  ${}^6\text{Li}$  nuclei, it is necessary to conduct studies in other channels, such as inelastic scattering, charge transfer and exchange channels. These studies will help test the generality of our results and provide a more complete description of the interactions and structural properties of nuclei.

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### Conflict of interest

The authors declare no conflict of interest.

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