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Matter distributions of light nuclei within the three-body model

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Matter distributions of light nuclei in the three body model $\left[{}^{6}\mathrm{He},{}^{6}\mathrm{Li}:(\alpha+\mathrm{n}+\mathrm{n})\ \text{and}\ {}^{9}\mathrm{Be}:(\alpha+\alpha+\mathrm{n})\right]$ is theoretically expressed. As a quantum state of the system was adopted a wave function based on the multi dynamic cluster model. Within the multi dynamical cluster model with Pauli projection, matter density distribution and matter rms radius for nuclei ^{6}He , ^{6}Li , ^{9}Be were analytically calculated. Comparisons of calculated matter radii with experimental data are presented.

Keywords: three-body system, matter distribution, variational method.

Introduction

Nucleon associations or so-called cluster structures of atomic nuclei have manifested themselves as a phenomenon when they were discovered in alpha decay. Consequently, in atomic nuclei there are not only nucleons moving independently of each other, but there are separate subsystems like alpha particles. In this way, having several types of simple structures one able to describe a number of atomic nuclei within the framework of the cluster model. A clear example of such systems can be both stable nuclei: ^{9}Be , ^{6}Li , ^{12}C , ^{16}O , and nuclei beyond the stability: ^{6}He , ^{9}He , ^{9}Li , ^{11}Li . These nuclei have very distinctive properties determined by the relative motion of clusters such as rms radii, binding energy, spectra of low lying excited states and so on. In a significant number of cases they find explanations only within the cluster model. The study of such problems remains one of the priority tasks due to the rapid development of experimental techniques for studying properties of atomic nuclei and with increasing interest in understanding the internal structure of the such kind of nuclei.

In the cluster model, the diagonalization of the matrix elements of the hamiltonian is greatly simplified by reduction of degrees of freedom. Such problems are successfully solved by a variational method, where they are used not only in nuclear physics, but also in particle physics, in solid-state physics and in atomic physics. There remains only a question of basis choice and interaction potentials.

The multi dynamic cluster model with the Pauli project [1] was successfully applied in studying the structure of nuclei: ^{3}He , ^{3}H , ^{6}He , ^{6}Li , ^{9}Be and ^{9}B , using the method of stochastic variations without introducing any free parameters. In this regard, it should be noted that the well known configurations: cigar-shaped and helicopter-like, were first discovered in this model for ^{6}He and ^{6}Li [2]. Also within the model a good description of many observables for the above-mentioned nuclei is given. In particular, low-lying excitation spectra, static characteristics, electromagnetic form factors and decay constants [3,4].

It would be interesting to know a value of matter radius, which was not subject of research of the multi dynamic cluster model. If it is included the cluster model in, then, one may come closer to understanding the structure of the nucleus. Objects are being researched are ^{6}He , ^{6}Li and ^{9}Be . The aim is to calculate theoretically matter rms of the latter, and compare it with other data sources. Since they have explicitly expressed cluster structure, it is also interesting to investigate matter density distribution. The article first gives a representation of the wave function of the three-particle system, then the theoretical expression of matter density distribution, and at the end results and discussions.

Theoretical model

The multi dynamic cluster model with Pauli project

The model [1] uses three pair pseudo-potentials to describe the three-body system, taking into account forbidden states by the Pauli principle:

$$\widetilde{V}_{ij} = V_{ij} + \Delta_{ij}, \tag{1}$$

where V_{ij} is an interaction potential of (ij) subsystem and $\Delta_{ij} = \lambda\Gamma$ is a orthogonalizer with the λ constant and with Γ projector, which for forbidden f state is as follows:

$$\Gamma = \Gamma(f) = \sum_{m_f} |\phi_{fm_f}(\mathbf{x})\rangle\langle\phi_{fm_f}(\mathbf{x}')|\delta(\mathbf{y}-\mathbf{y}'). \tag{2}$$

The principle plays a huge role in the structure of the nucleus, which does not allow overlapping of two constituent particles. Thus, the three body pseudo-Hamiltonian including kinetic energy and pseudo potentials looks like this

$$\widetilde{H} = H_0 + \sum_{i<j} \widetilde{V}_{ij}. \tag{3}$$

The trial function is given in the form:

$$\Psi_{JM_JTM_T} = \sum_{\imath=1}^{N} C_\imath \varphi_\imath^\gamma(1,23) \tag{4}$$

with expansion coefficients $C_\imath$ and dimension N. A basis function $\varphi_\imath^{\widetilde{\gamma}}(i,jk)$ is taken by a product of space, spin and isospin parts

$$\varphi_\imath^\gamma(i,jk) = [\Phi_L^{\lambda,l}(i,jk) \times \chi_S^s(i,jk)]_{JM_J}\tau_{TM_T}^t(i,jk), \tag{5}$$

where γ is a set containing all of moments. Spatial part $\Phi_L^{\lambda,l}(i,jk)$ is constructed with the Gaussian

$$\Phi_L^{\lambda,l}(i,jk) = x_i^\lambda y_i^l \exp(-\alpha x_i^2 - \beta y_i^2)[Y_\lambda(\widehat{x}) \times Y_l(\widehat{y})]_{LM_L}, \tag{6}$$

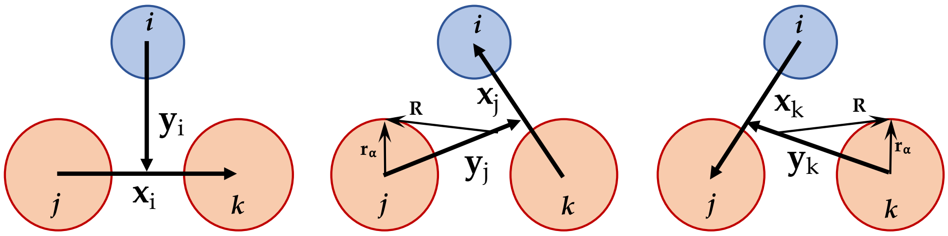

Figure 1. Example of the three body system of ^{9}Be on relative Jacobi coordinate sets with cyclically particle permutations. The last two sets include the scheme of taking into account a size $\mathbf{r}_\alpha$ of sub-alpha particles.

where λ is a orbital moment of the jk pair conjugated to the x Jacobi coordinate, while l is the moment of i spectator conjugated to the y relative coordinate (see Figure 1) and α, β are non linear parameters.

The basis function (5) is convenient for the ability to convert into other sets of relative Jacobi coordinates. In particular, in the spatial part a transformation of the set i set into the set j is

$$\Phi_L^{\lambda,l}(i,jk) = \sum_{\widetilde{\gamma}} A_\Omega^{j\leftarrow i} \Phi_L^{\widetilde{\lambda},\widetilde{l}}(j,ki), \tag{7}$$

where the sum is constrained by $\lambda + l = \widetilde{\lambda} + \widetilde{l}$ condition. A recouple coefficient $A_\Omega^{j\leftarrow i}$ with rotation matrix $\Omega_{j\leftarrow i}$ is

$$A_\Omega^{j\leftarrow i} = (-1)^{\lambda+l} \sum_{\lambda_1,\lambda_2,l_1,l_2} \Omega_{11}^{\lambda_1}\Omega_{12}^{\lambda_2}\Omega_{21}^{l_1}\Omega_{22}^{l_2} \sqrt{\frac{[\lambda]![l]![\lambda][l][\lambda_1][\lambda_2][l_1][l_2][\widetilde{\lambda}][\widetilde{l}]}{[\lambda_1]![\lambda_2]![l_1]![l_2]!}} \times \\ \begin{pmatrix} \lambda_1 & \lambda_2 & \lambda \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l_1 & l_2 & l \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \lambda_1 & \lambda_2 & \lambda \\ l_1 & l_2 & l \\ \widetilde{\lambda} & \widetilde{l} & L \end{pmatrix}, \tag{8}$$

where $[x] = 2x+1$, $\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$ and $\begin{pmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \\ j_7 & j_8 & j_9 \end{pmatrix}$ are the Wigner 3-j and 9-j symbols consequently.

Matter density distribution

An operator of matter density distribution of the three body system takes the sums of all three clusters and brings it from the center of system mass

$$\rho(\mathbf{R}) = \sum_{cluster=i,j,k} \rho_{cluster}(\mathbf{R}). \tag{9}$$

In particular, nucleon cluster is treated as point like particle, while in alpha cluster one takes into account its internal structure (see Figure 2) $\rho_\alpha(r_\alpha) = \rho_0 \exp\left(-\gamma_0 r_\alpha^2\right)$ with parameters $\gamma_0 = 0.7024$, $\rho_0 = 0.4229$ [5]. So corresponding matrix elements for both nucleon cluster and alpha cluster are

$$\begin{aligned} \rho_{N_i}(\mathbf{R}) &= \langle \varphi^\gamma(i,jk)|\delta(\mathbf{R}-\mathbf{y}_i)|\varphi^\gamma(i,jk)\rangle \\ \rho_{\alpha_i}(\mathbf{R}) &= \langle \varphi^\gamma(i,jk)|\rho_\alpha(\mathbf{R}-\mathbf{y}_i)\delta(\mathbf{y}_i-\mathbf{r}_\alpha)|\varphi^\gamma(i,jk)\rangle \end{aligned} \tag{10}$$

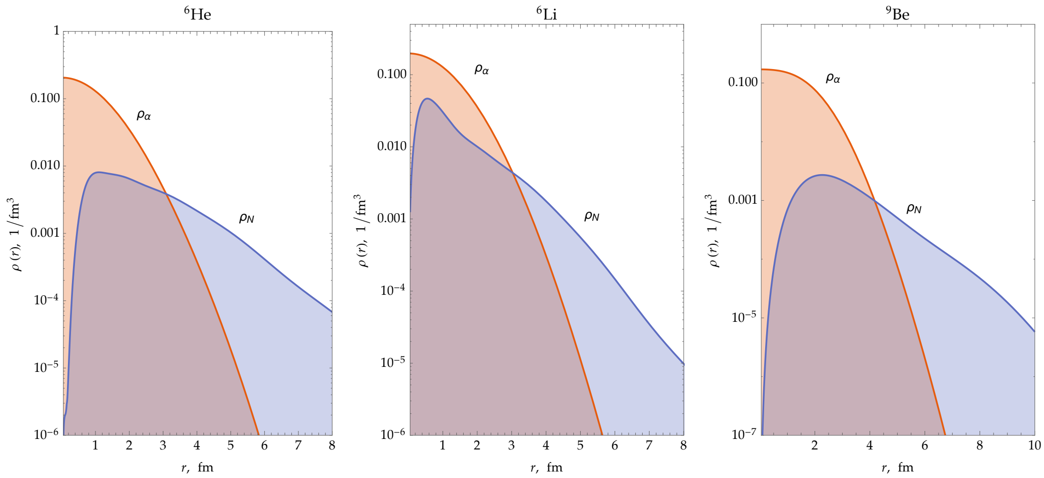

Figure 2. Matter density distribution of nuclei ^{6}He , ^{6}Li and ^{9}Be .
The sum of functions for each nucleus is normalized to it's atomic number.

In the case of the cluster is in another set of coordinate, one can express taking the equation (5)

$$\rho_{N_j}(\mathbf{R}) = \sum_{\widetilde{\gamma}} A_{\Omega}^{j\leftarrow i} A_{\Omega}^{j\leftarrow i} \langle \varphi^{\widetilde{\gamma}}(j,ki)|\delta(\mathbf{R}-\mathbf{y}_j)|\varphi^{\widetilde{\gamma}}(j,ki)\rangle$$

$$\rho_{\alpha_j}(\mathbf{R}) = \sum_{\widetilde{\gamma}} A_{\Omega}^{j\leftarrow i} A_{\Omega}^{j\leftarrow i} \langle \varphi^{\widetilde{\gamma}}(j,ki)|\rho_\alpha(\mathbf{R}-\mathbf{y}_j)\delta(\mathbf{y}_j-\mathbf{r}_\alpha)|\varphi^{\widetilde{\gamma}}(j,ki)\rangle.$$

Using the well known expansion of exponent function

$$\exp(-\rho_0 \mathbf{R}\cdot\mathbf{y}) = 4\pi \sum_k \sqrt{2k+1}\; i_k(\rho_0 R y) Y_{00}^{kk}\left(\widehat{R},\widehat{y}\right)$$

and an analytical expression of integral kind of

$$\int_0^\infty y^{2l+k+2}\exp(-\beta y^2) i_k(\mu y)dy = \sqrt{\frac{\pi}{2}}\frac{(2l)!!(\mu)^k}{\left(\frac{1}{2}\beta\right)^{l+k+3/2}}\exp\left(\frac{\mu^2}{\beta}\right) L_l^{k+\frac{1}{2}}\left(-\frac{\mu^2}{\beta}\right)$$

one able to obtain equation (10) analytically for both nucleon and alpha clusters as follows

$$\rho_{N_i}(R) = \frac{1}{2}\left(\frac{R}{y_0}\right)^{2l+2}\Gamma\left(\frac{3}{2}+\frac{\lambda}{2}\right)\sum_{\imath\jmath} C_\imath C_\jmath \frac{\exp\left(-\frac{(\beta_\imath+\beta_\jmath)}{y_0^2}R^2\right)}{(\alpha_\imath+\alpha_\jmath)^{\frac{3}{2}+\frac{\lambda}{2}}}$$

$$\rho_{\alpha_i}(R) = (2\pi)^{\frac{3}{2}}\rho_0 \sum_{\imath\jmath} C_\imath C_\jmath \frac{\Gamma\left(\frac{3}{2}+\lambda\right)\;(2l)!!\;L_l^{1/2}\left(-\frac{(\gamma_0)R^2}{\beta_\imath+\beta_\jmath+\gamma_0}\right)}{(\alpha_\imath+\alpha_\jmath)^{\frac{3}{2}+\lambda}\;(\beta_\imath+\beta_\jmath+\gamma_0)^{\frac{3}{2}+l}}\times \qquad (11)$$

$$\exp\left(\left(-\gamma_0+\frac{\gamma_0^2}{\beta_\imath+\beta_\jmath+\gamma_0}\right)\left(\frac{R}{y_0}\right)^2\right)$$

where $y_0 = \frac{m_j+m_k}{m_i+m_j+m_k}$, $i_k(x)$ - modified spherical Bessel function of the first kind, $L_l^{k+1/2}(x)$ - associated Laguerre polynomial and $\Gamma(x)$ - Gamma function.

Results and discussions

Energy minimization processes was carried out with BFW [6], the modified SBB [3] and RSC [7] potentials for $\alpha\alpha$-, αn- and nn- subsystems, respectively. The attracting BFW potential with forbidden states is parametrized in gaussian form and describes the experimental phase shifts well. The modified SBB potential also excludes forbidden states, describes the phase shifts of the scattering, moreover, takes into account even-odd splittings. All of findings for nuclei ^{6}He, ^{6}Li and ^{9}Be are presented in Table 1. For a detailed discussions of similar results of ground state properties see [3,4].

In a Figure 2 matter density distributions of each cluster of nuclei ^{6}He, ^{6}Li and ^{9}Be are shown. It should be noted that the sum of cluster density distributions is normalized to its nucleus atomic number. A feature of function behavior of nucleonic density component is extended tail, and at small distances peculiar minimum (except of ^{6}Li). These facts reflect that nucleons are moving away from the center of mass, and are extremely weakly bound, thereby confirming the cluster structure this nuclei. As for the nucleon density of lithium at a small distance, an insignificant minimum is due to the high binding energy of the nucleus.

Table 1.
Ground state property and comparison of calculated matter radii with experimental data

	^{6}He	^{6}Li	^{9}Be
E_b, MeV (this work)	0.23	3.26	1.58
E_b, MeV	0.97	3.7	1.57
r_{mat}, fm (this work)	2.97	2.447	2.62
r_{mat}, fm	2.62 [8]	2.45 [9]	2.61 [10]*

*The data was based on the AMD model

Matter rms radii of nuclei ^{6}He, ^{6}Li and ^{9}Be were also calculated. Results in comparison with other works is given in Table 1. An overestimation of calculated matter rms with the experimental value revealed for ^{6}He. In this instance, it should be pointed out that the theoretical value of the binding energy of ^{6}He within the framework of the multi dynamical cluster model also had a distinction with the experimental value to the difference 0.7 MeV. Perhaps, a basis of revaluation rests on a wave function which does not take into account the symmetry and is in incompleteness of the variational basis. Values of the matter rms of ^{6}Li and ^{9}Be are in good agreement with other sources.

Conclusion

Within the multi dynamical cluster model with Pauli projection, matter density distribution and matter rms radius for nuclei ^{6}He, ^{6}Li, ^{9}Be were analytically calculated. The behavior of these functions was explained. Also, comparisons of radii with other sources are given. Theoretical calculations are in good agreement with given data, except for the ^{6}He nucleus. By and large, submitted theoretical method of calculating matter density distribution makes possible to understand the internal structure of the three body system.

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